Causality Aspects of the Parton Cascade Approach to Ultarelativistic Heavy Ion Reactions †

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Introduction

Parton cascade codes that take a space-time approach to model the microscopic processes [1] of an ultrarelativistic heavy-ion interaction are – in spite of their QCD bells and whistles – by necessity based on some kind of *classical many-particle dynamics*.

Problems:

- space-time-based cascade models suffer from the consequences of the

No-Interaction Theorem (NIT) [CJS63]:

> The only consistent many-particle Hamiltonian theory that is Poincaré-covariant is that of a system of free particles.

- the procedure to determine the sequence of the binary parton interactions ("SBPI") – an essential aspect of space-time-based cascade models – is by necessity an artificial and ad-hoc feature of these codes.

The only way to circumvent the NIT is to loosen its assumptions:

1. forget about Poincaré covariance $\Longrightarrow$ **VNI**

   In this approach the SBPI depends on the initially chosen frame of reference. Einstein causality remains a problem.

2. introduce a many-times formalism, e.g. by formulating the model in $8N$-dimensional phase space (N is the particle number) [2] $\Longrightarrow$ **pcpc**

   The Poincaré covariance of this model seems to guarantee Einstein causality; the SBPI of the code, however, deserves closer scrutiny.

[1] Prominent examples are **VNI** and **pcpc**; cf. the OSCAR archive at `http://nt3.phys.columbia.edu/people/molnard/OSCAR/` .

[2] We have shown previously that for $N = 2$ all known Poincaré-covariant formalisms are equivalent to ours (cf. [PNB94]).

Basic Covariant Structure of pcpc

pcpc is a hybrid of a *classical* dynamics approach that governs the evolution of the system between binary parton interactions, and a parton interaction model with QCD ingredients (parton distribution functions, pQCD cross sections and 'DGLAP evolution') [3].

- The dynamical evolution of the system is parametrized by a Poincaré scalar, s (in contrast to the usual t, the proper time of an external observer, as measured in some external frame)

- The phase-space variables of the N partons (quarks, gluons) are covariant 4-vectors ${x_i}^{\mu}(s)$, ${p_i}^{\mu}(s)$. Between binary interactions N is fixed, but can (and does) vary due to parton creation in the QCD-governed parton interactions

- The ($8N$-dimensional) interaction term of the Poincaré-*invariant* 'Hamiltonian' depends on the following Poincaré-invariant '4-distances'

$$ {d_{ij}}^{2}(s) := -\hat{x}^{\,2} = -(x\hat{x}) = -(\hat{x}x) $$

$$ \hat{x}^{\,\mu} := x^{\,\mu} - \frac{(xp)}{p^2} p^{\,\mu} $$

  [$x^{\,\mu}(s) := {x_i}^{\mu}(s) - {x_j}^{\mu}(s), p^{\,\mu}(s) := {p_i}^{\mu}(s) + {p_j}^{\mu}(s)$ are the relative 4-distance and total 4-momentum of particles i and j]

  The physical reasons why in Poincaré-covariant dynamics the interactions can only depend on these "orthogonal projections" $\hat{x}^{\mu}$ have been given before [PNB94].

- Binary parton interactions occur at the s determined by ${d_{ij}}^{2}(s)$ being at a minimum [4] (in the cms of partons i, j, ${d_{ij}}^{2}$ is the minimal 3-distance of approach)

- Between interactions, partons move along free trajectories:

$$ {x_i}^{\mu}(s) = \frac{{p_i}^{\mu}(s)}{m_i}(s - s_0) + {x_i}^{\mu}(s_0) $$

[3] For details, cf. [PNB94, BMGMN00]

[4] The interaction term in the Hamiltonian thus can be thought of as a sum of δ-functions.

Logical flow in pcpc

In contrast to most other cascade codes, the 'time step' in **pcpc** is not a computational artifice, but is given by the formalism itself:

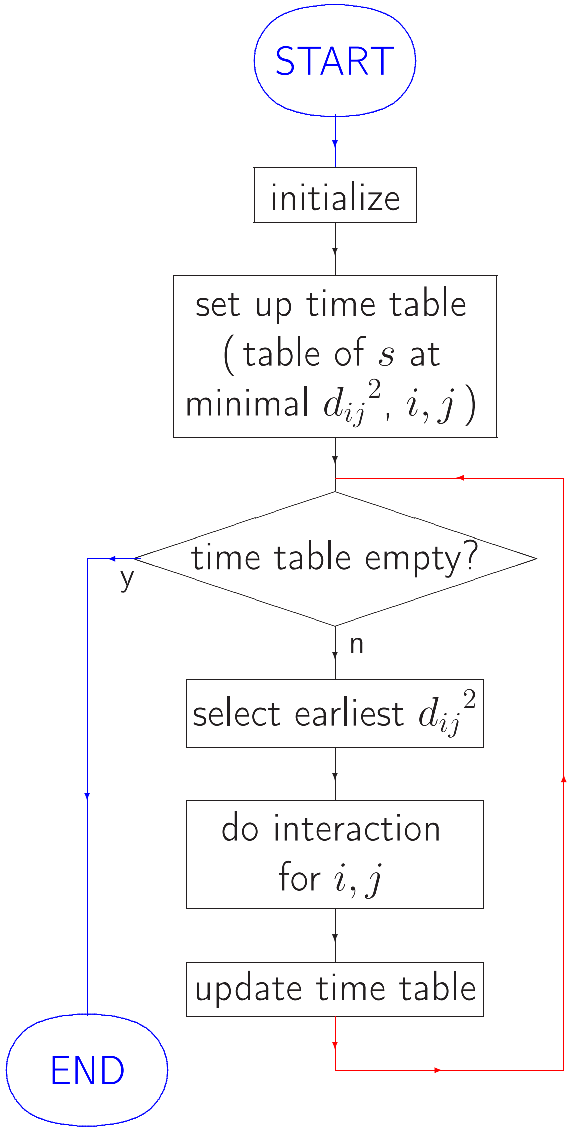
- if there were only 2 partons, their interaction would occur at the value of the evolution parameter s at which they would reach their minimal ${d_{ij}}^2(s)$ if they were indeed alone in the world,

- so a 'time table' is kept, containing, for every pair i, j of partons, its potential minimal approach ${d_{ij}}^2(s)$ (i.e. their minimal approach if i, j were the only partons in the whole system), and the corresponding s,

- then this table is searched for the *smallest* ('earliest') s. At this s the potential interaction of the corresponding pair *will actually occur* (because all other potential interactions are 'later'),

- the interaction of the pair i, j will change the world lines of these two partons (and possibly create further partons). Therefore, the time table is updated, with new values of ${d_{ij}}^2(s)$ for all pairs involving either parton i or parton j (or the newly created partons); and the code loops.

Thus the code follows exactly the sequence of binary parton interactions as parametrized by the monotonically increasing *Poincaré-invariant* evolution parameter s. The logic is summarized in the following flow diagram:



Do the parton interactions preserve Einstein causality?

Prima facie this does not seem to be so: the 4-vector $x_i{}^\mu - x_j{}^\mu$ is space-like, and so no signal can be transmitted between events x_i and x_j. But this argument is fallacious: while it would be correct in the frame work of a $6N$-dimensional phase space formalism and physical observer time, it does not follow in a many-times formalism with an $8N$-dimensional phase space.

Furthermore, it must be remembered that the representation of the (QCD) physics of the heavy-ion reaction in terms of cross sections, parton distribution functions etc. is tantamount to the description of intrinsically quantum processes in a *classical* terminology. But as long as we refrain from trying to look *inside* an individual binary parton interaction with classical concepts, Einstein causality is not infringed upon by such a model (for details on this point cf. [PNB94]).

Is Einstein causality preserved between binary interactions?

In terms of the cascade picture, a heavy-ion reaction can be seen as one or several disjoint graphs of connected particle world lines. In every connected subgraph, the Poincaré-covariant dynamics guarantees Einstein causality.

Separate disjoint subgraphs, however, can have no causal connection. But by construction, no signals are transmitted between them. Imagining a full quantum-mechanically description of such a system, unconnected subgraphs would correspond to subamplitudes which would simply be multiplied in obtaining the total amplitude.

Are the initial parton *positions* critically important?

There is no physically convincing argument for how to set the time components of the initial parton 4-vectors $a_i := {x_i}^{\mu=0}(s = s_0)$; so these must be fixed phenomenologically with some arbitrary prescription. In **pcpc** they are all set to zero.

Does this imply the choice of a particular initial frame of reference, thus invalidating covariance, or spoiling the Einstein causality of the model? The answer is

NO!

To see this, suppose that we *do not* set the initial $a_i = 0$, but retain them as free (arbitrary) parameters. We would then find the minimum of the ${d_{ij}}^2$ to be formally dependent on the a_i. But since for any two time-like 4-vectors the invariant quantity $(x\hat{y})$ is simply $-\vec{x}\cdot\vec{y}\Big|_{\text{cms}}$, we find that the

${d_{ij}}^2$ are actually *independent of the* (time components of the) *initial parton positions*. It follows that the parton interactions are implemented in a Poincaré-covariant way, even though their sequence (SBPI) *does* depend on how the initial parameters are chosen.

Conclusions

To sum up, causality is *not an issue* in discussing the physical validity of a cascade model that uses a space-time approach to the dynamical evolution (provided the model is Poincaré-covariant).

It is, however, important to realize that in such codes the sequence of binary parton interactions (SBPI) is to be considered an essential *part of the model*, and that it is *necessarily phenomenological* in character. In Poincaré-covariant cascade codes, such as **pcpc**, the SBPI (although remaining a phenomenological prescription) is independent of choice of the frame of reference in which the code is run.

The difficulties with Einstein causality incurred by non-covariance of the SBPI have been discussed many years ago [KDCDN84]. In contrast to the situations described in that paper, Einstein causality is preserved in a Poincaré-covariant model such as **pcpc**, both for the individual binary interactions and for the dynamic evolution of the system as a whole.

References

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